For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

1) \(y = -x^2, \ y = -1, \ x = 0, \ x = 1\)  
Axis: \(y = -1\)

2) \(y = \sqrt{x - 1}, \ y = -1, \ x = 1\)  
Axis: \(y = -1\)

3) \(y = x^2 + 2, \ y = -1, \ x = -1, \ x = 1\)  
Axis: \(y = -1\)

4) \(y = -x^2 + 3, \ y = 2\)  
Axis: \(y = 2\)
For each problem, use the method of cylindrical shells to find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

5) \( y = -x^2 + 4, \ y = x^2 + 2, \ x = -1, \ x = 0 \)  
Axis: \( x = -1 \)

6) \( y = -x^2 + 6, \ y = 2, \ x = 0, \ x = 2 \)  
Axis: \( x = 3 \)

7) \( y = x^2 + 1, \ y = 1, \ x = 2 \)  
Axis: \( x = -4 \)

8) \( y = \sqrt{x} + 1, \ y = x^2 + 1 \)  
Axis: \( x = -2 \)
For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

1) \( y = -x^2, \ y = -1, \ x = 0, \ x = 1 \)
   Axis: \( y = -1 \)
   \[
   \pi \int_{-1}^{1} (-x^2 + 1)^2 \, dx = \frac{8}{15} \pi \approx 1.676
   \]

2) \( y = \sqrt{x} - 1, \ y = -1, \ x = 1 \)
   Axis: \( y = -1 \)
   \[
   \pi \int_{-1}^{1} (\sqrt{x})^2 \, dx = \frac{1}{2} \pi \approx 1.571
   \]

3) \( y = x^2 + 2, \ y = -1, \ x = -1, \ x = 1 \)
   Axis: \( y = -1 \)
   \[
   \pi \int_{-1}^{1} (x^2 + 3)^2 \, dx = \frac{112}{5} \pi \approx 70.372
   \]

4) \( y = -x^2 + 3, \ y = 2 \)
   Axis: \( y = 2 \)
   \[
   \pi \int_{-1}^{1} (-x^2 + 1)^2 \, dx = \frac{16}{15} \pi \approx 3.351
   \]
For each problem, use the method of cylindrical shells to find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis.

5) \( y = -x^2 + 4, \ y = x^2 + 2, \ x = -1, \ x = 0 \)
   Axis: \( x = -1 \)

\[
2\pi \int_{-1}^{0} (x + 1)(-x^2 + 4 - (x^2 + 2)) \, dx
\]

\[
= \frac{5}{3}\pi
\]

6) \( y = -x^2 + 6, \ y = 2, \ x = 0, \ x = 2 \)
   Axis: \( x = 3 \)

\[
2\pi \int_{0}^{2} (3 - x)(-x^2 + 6 - 2) \, dx
\]

\[
= 24\pi
\]

7) \( y = x^2 + 1, \ y = 1, \ x = 2 \)
   Axis: \( x = -4 \)

\[
2\pi \int_{-4}^{0} (x + 4)(x^2 + 1 - 1) \, dx
\]

\[
= \frac{88}{3}\pi
\]

8) \( y = \sqrt{x} + 1, \ y = x^2 + 1 \)
   Axis: \( x = -2 \)

\[
2\pi \int_{-2}^{1} (x + 2)(\sqrt{x} + 1 - (x^2 + 1)) \, dx
\]

\[
= \frac{49}{30}\pi
\]